Complex Networks in Evolutionary Computation and Heuristic Search

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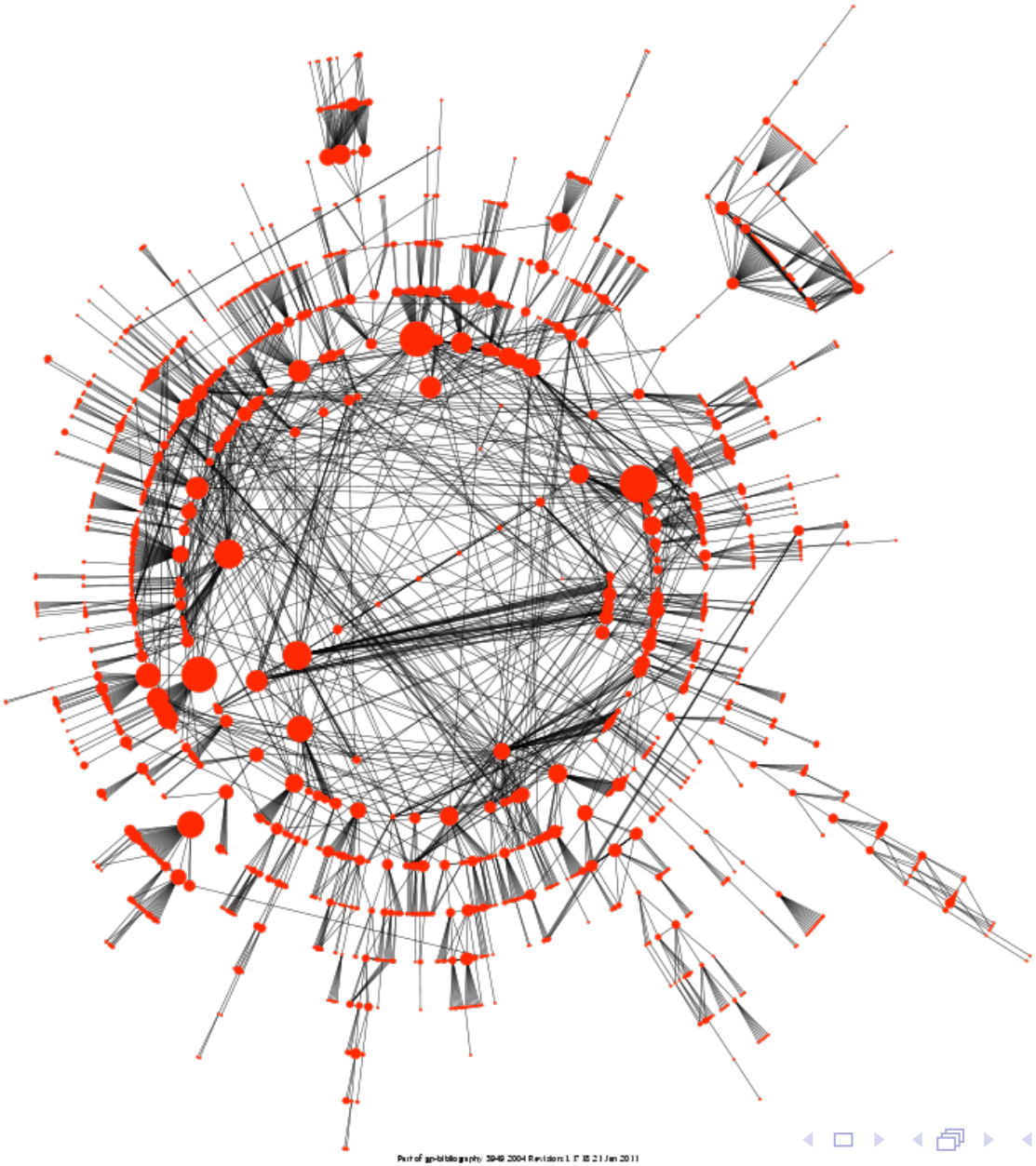
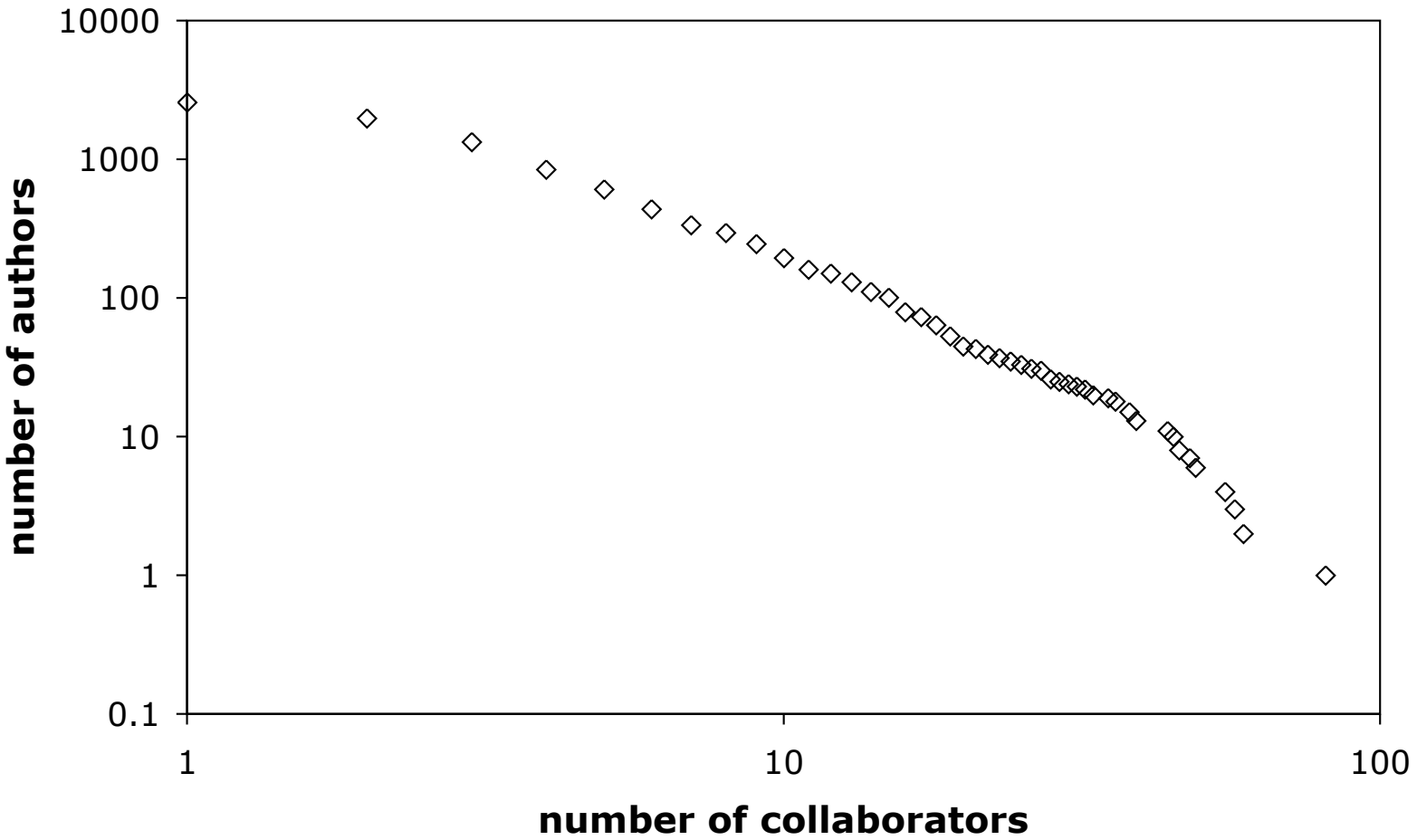
- They are clustered: locally many triangles and polygons; at the mesoscopic scale: many of them, especially social ones, have communities
- Their degree distribution functions $P(k)$ are often right-skewed: stretched exponentials, power-laws $P(k) \propto k^{-\gamma}$

The GP collaboration graph (B.W. Langdon)

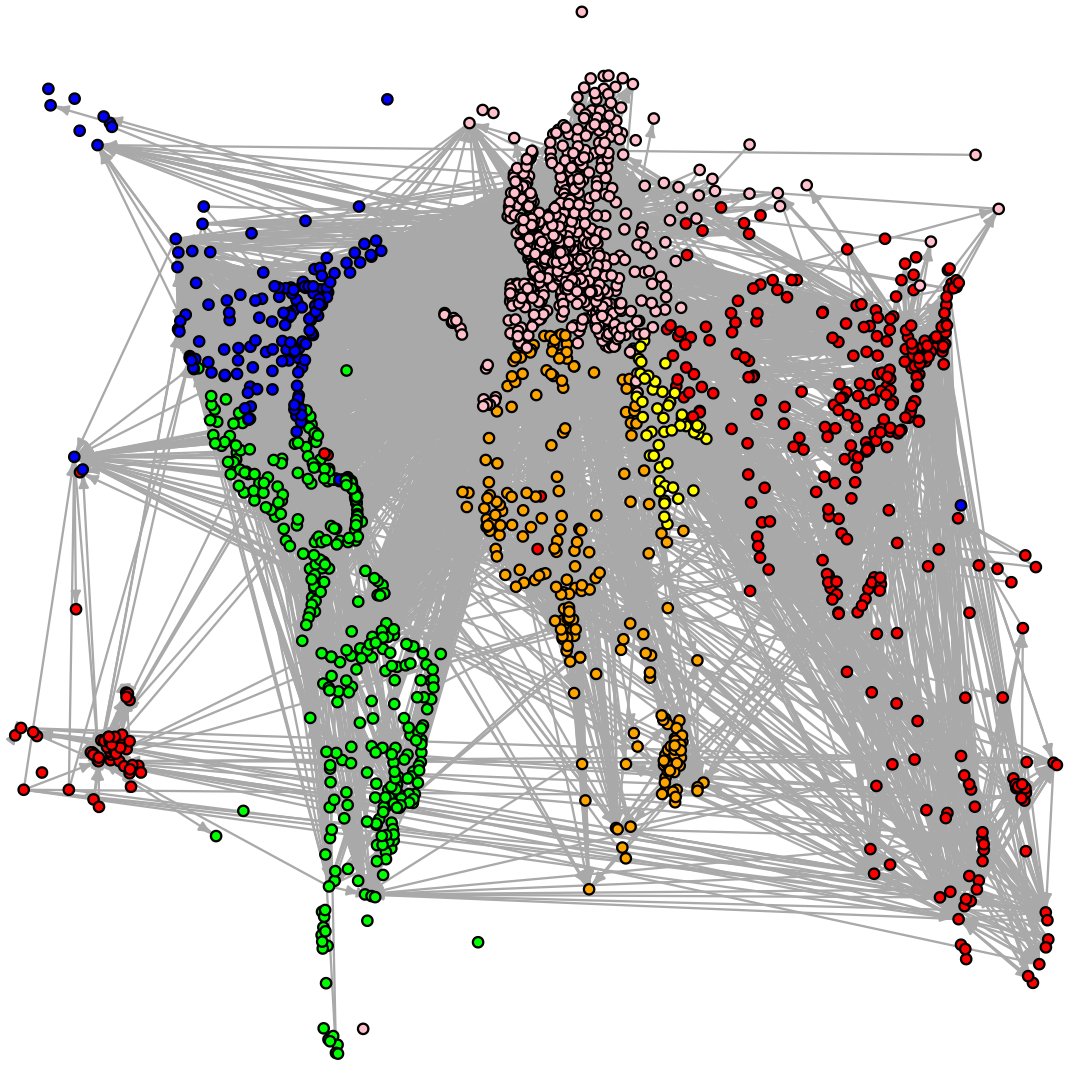


The GP collaboration graph

Cumulative Degree Distribution

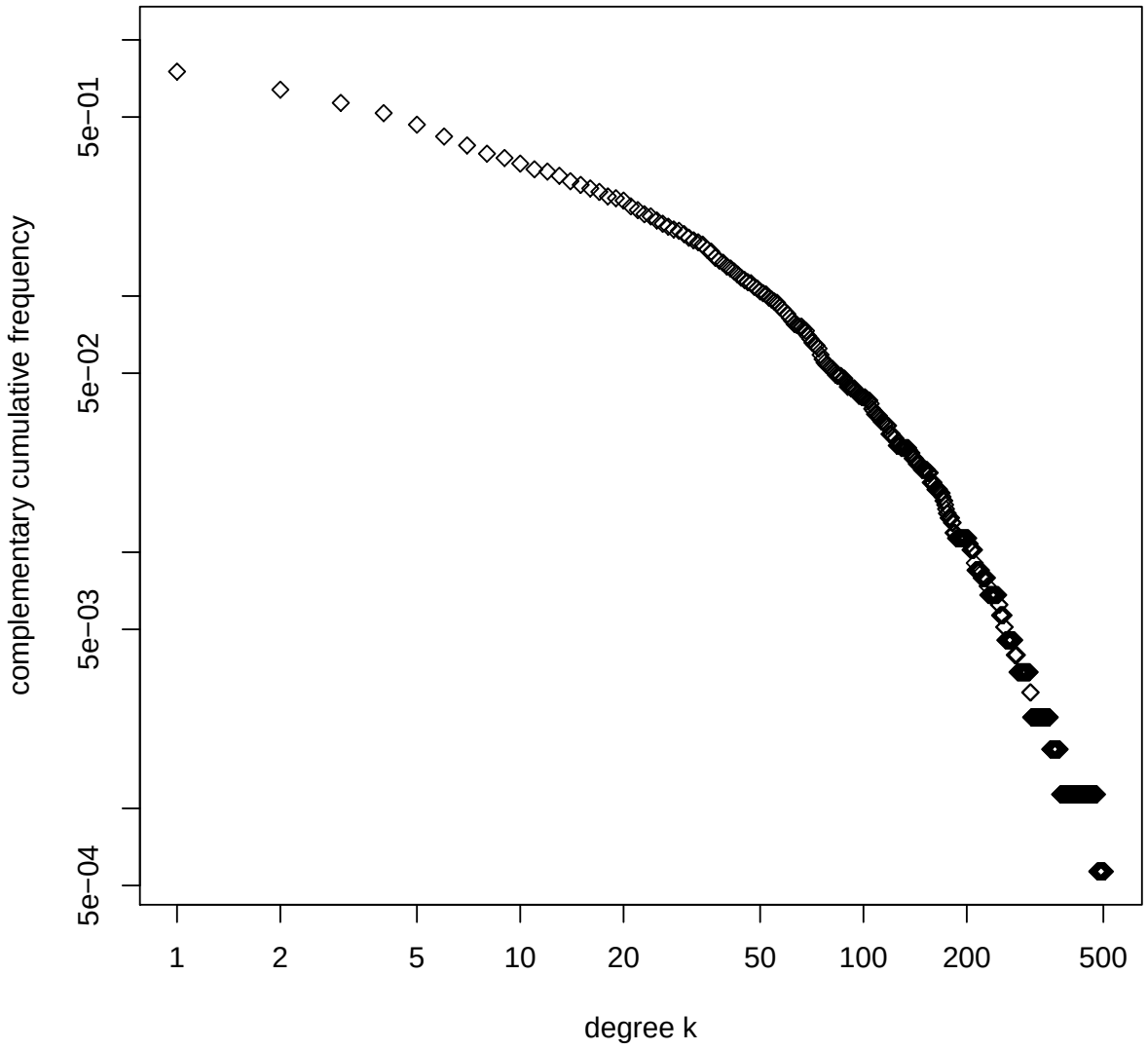


The World Airlines Graph (2000))

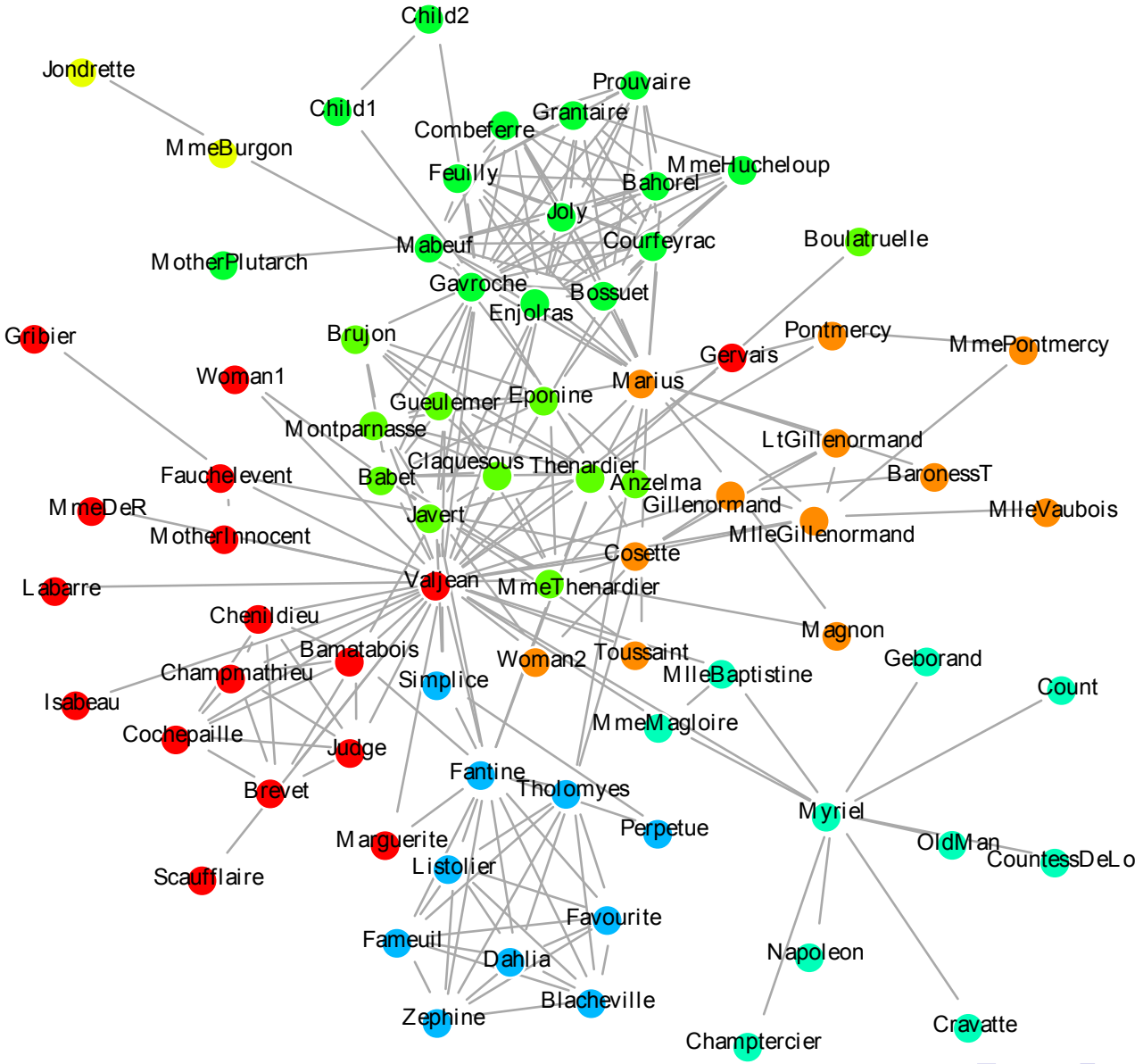


(courtesy of C. Rozenblat et al., 2006)

World Airlines: Cumulative DDF



Communities: Les Misérables



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  - the clustering coefficient $\bar{C} \to 0$ as $N \to \infty$
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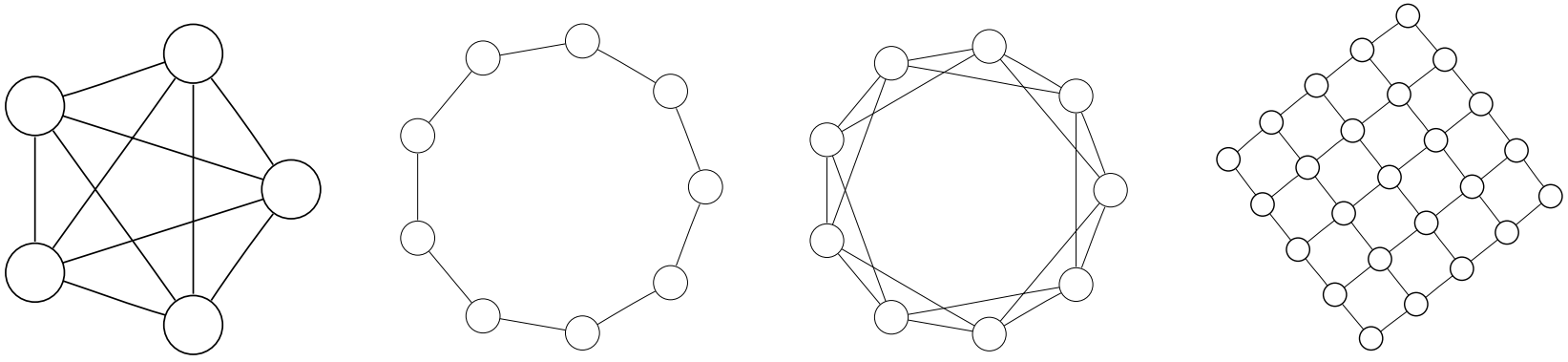
These models are unrealistic but very useful as statistical null models

Regular Graphs

In regular graphs all the nodes have the same degree:



Grids are often used but they are not good models for real networks: long paths, the same number of contacts for everybody

Model Complex Networks

Beyond random and regular graphs, including complete graphs, there exist today several new models that are much more faithful to actual network topologies.

Here we shall describe the two oldest and classical:

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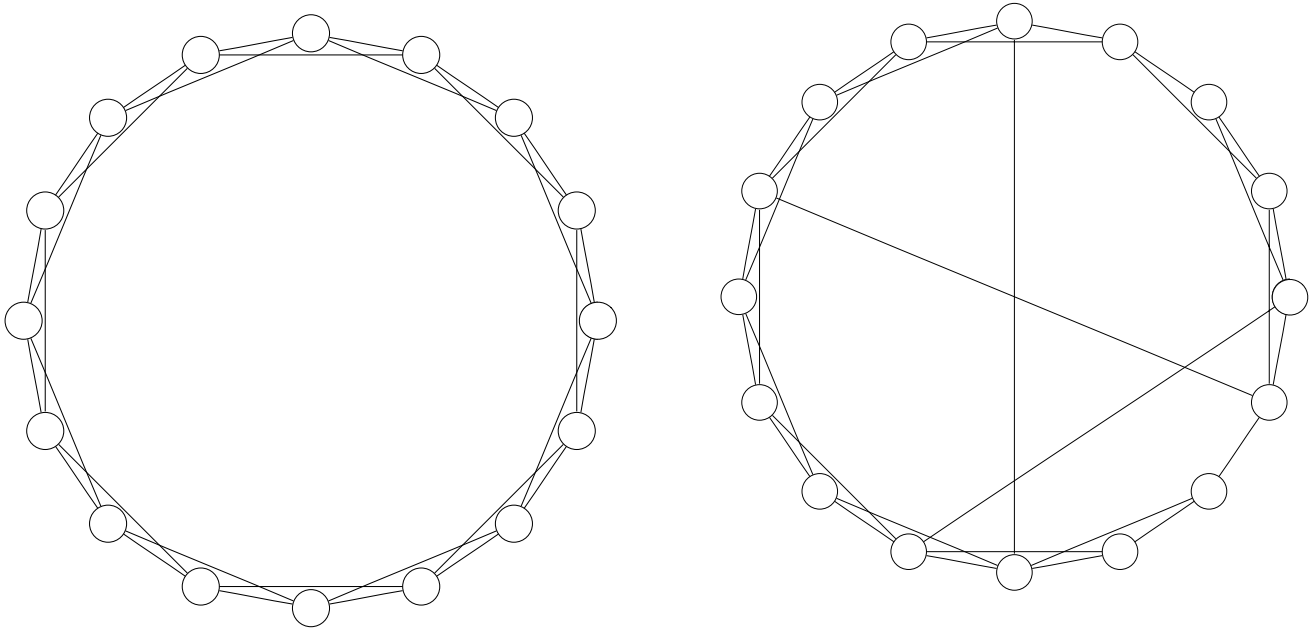
Here we shall describe the two oldest and classical:

- Watts–Strogatz networks
- Barabási–Albert networks

Watts–Strogatz Networks

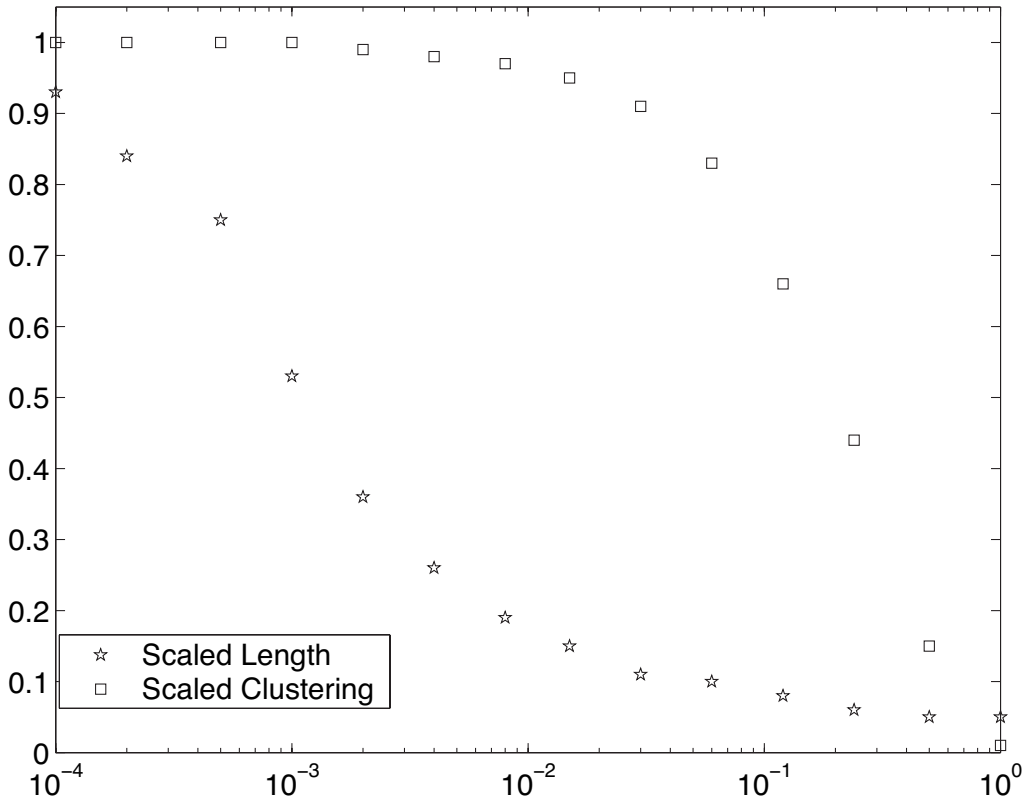
Go iteratively through each node and rewire each link with some small probability β to a randomly chosen node



"Shortcuts" shorten the mean path length by a large amount.

Watts–Strogatz Model: CC and Mean Path Length

Small-World region: short path length and high clustering coefficient



The $P(k)$ remains Poissonian however, no degree-heterogeneity

Redrawn from Watts and Strogatz '98

Barabási–Albert Growing Scale-Free Networks

- Start with a small clique (a completely connected graph) of N_0 nodes and M_0 edges.

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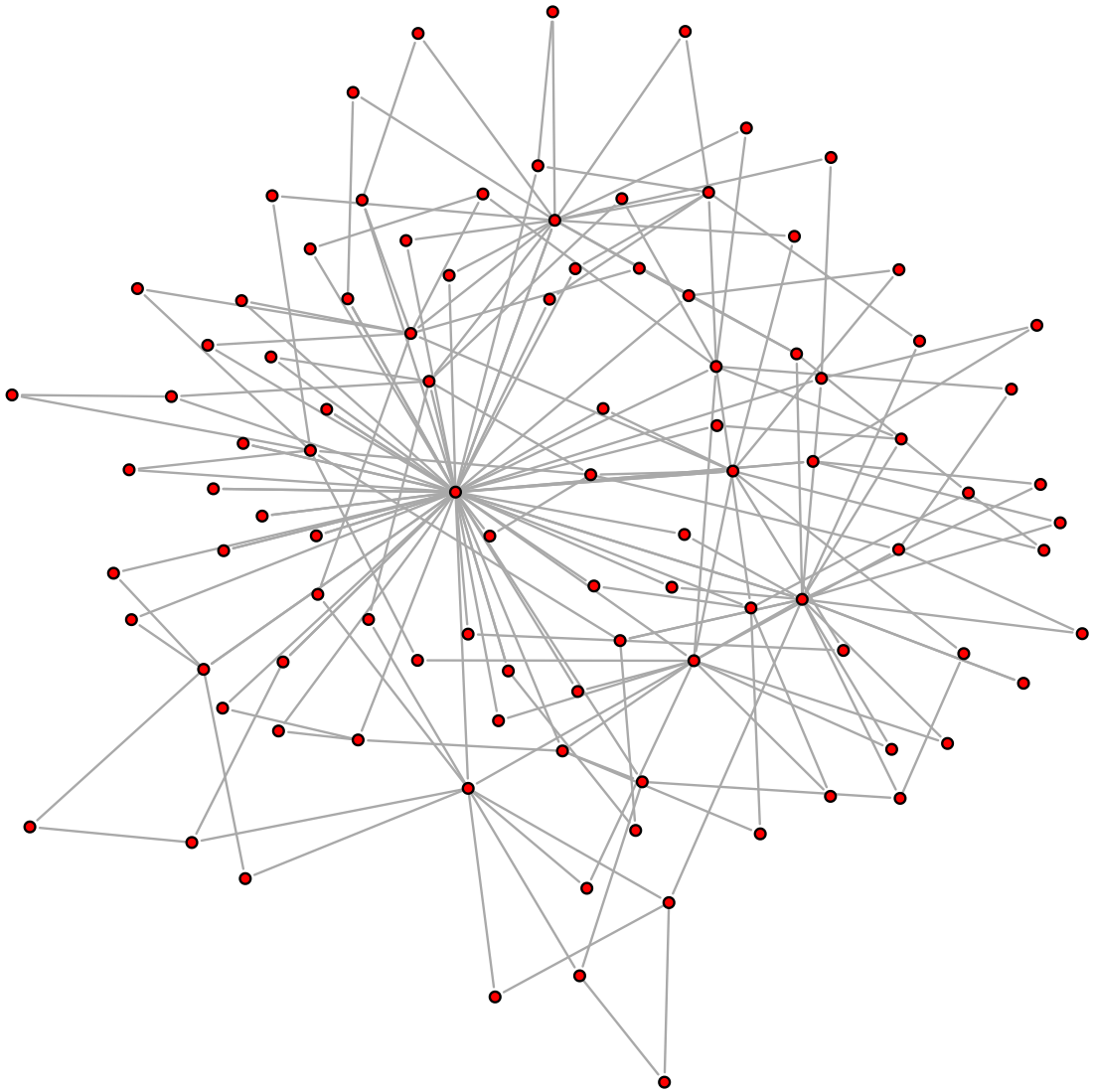
  - the probability π that a new node will be connected to node i is such that highly connected nodes are more likely to be chosen: $\pi(k_i) = \frac{k_i}{\sum_j k_j}$
- such a growing graph evolves into a stationary scale-free network with a power-law degree probability distribution $P(k) \sim k^{-\gamma}$, with $\gamma \sim 3$.

A Small Computer-Generated BA Scale-Free Network



Barabási–Albert Networks: Properties

- Barabási–Albert networks are based on preferential attachment which is an effect that has been observed in real networks, for example in references to web pages or to fundamental scientific articles among many others.
- They are much better than WS networks as models of real graphs
- They are very robust against random attacks
- However, they lack clustering and communities

Some references for this part

1. M. E. J. Newman, Networks: An Introduction, Oxford University Press, 2010 (The most complete)

2. G. Caldarelli, Scale-Free Networks, Oxford University Press, 2007 (Easier)

3. M. E. J. Newman, The structure and function of complex networks, SIAM Review, 45, 167-256, 2003. (Shorter)

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- population structures, selection pressure, and evolutionary algorithms
- network representation of search spaces

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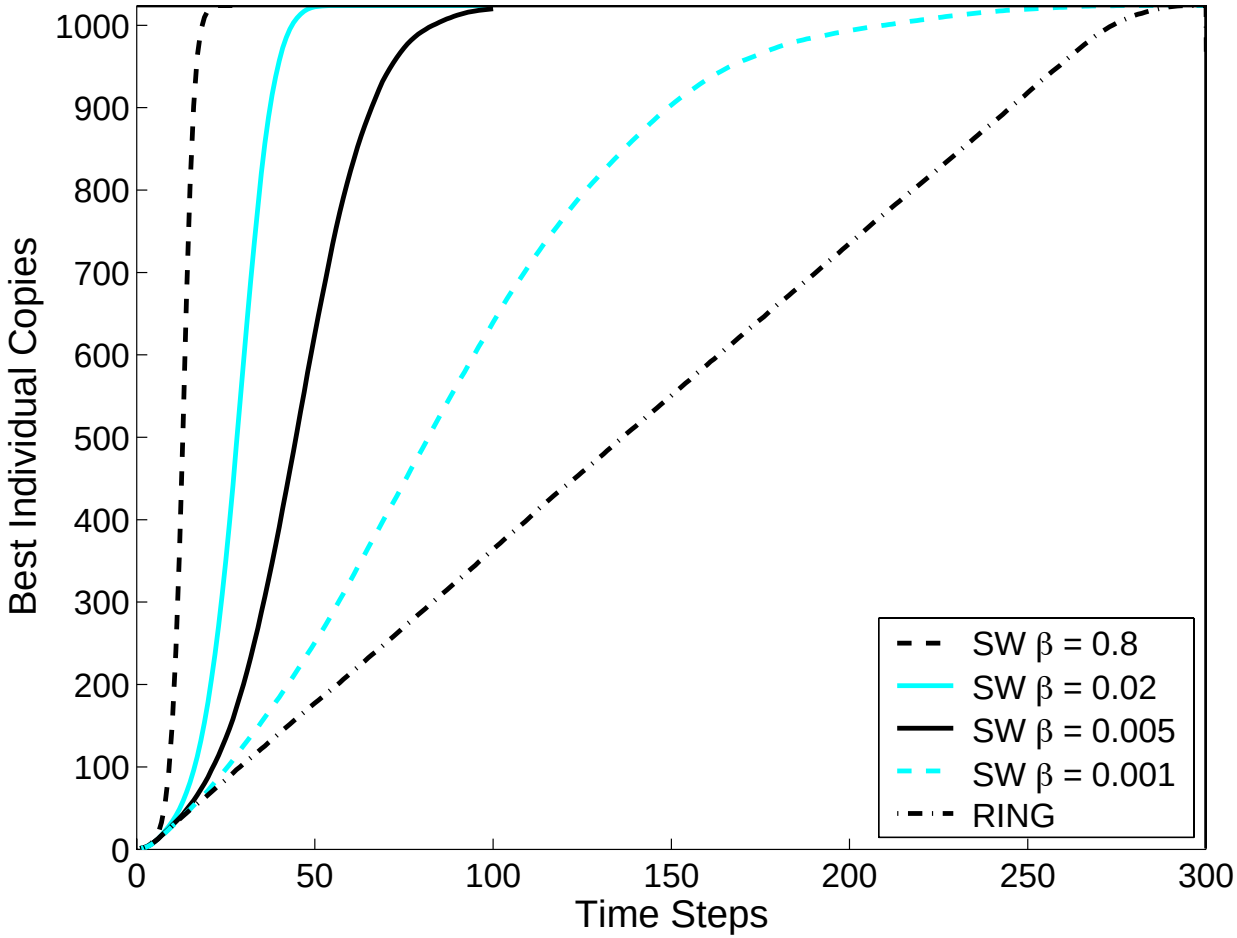
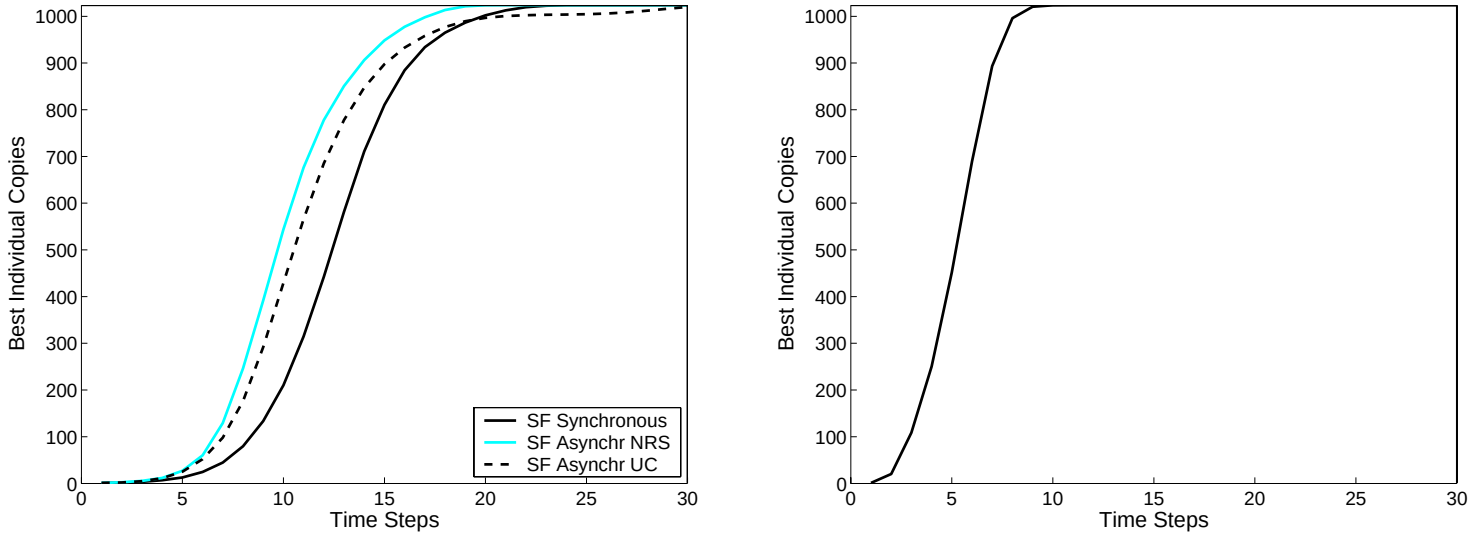
- cellular populations have always been low-degree regular graphs such as rings and toroidal grids
- here we shall explore other topologies drawn from the world of complex networks
- in evolutionary algorithms social or biological credibility of the contact structure is not strictly required and thus we can use what fits us best

Takeover Times on Watts–Strogatz Networks



One can indeed control the selection pressure (here binary tournament) from linear (ring) to exponential (random graph and well mixed population)

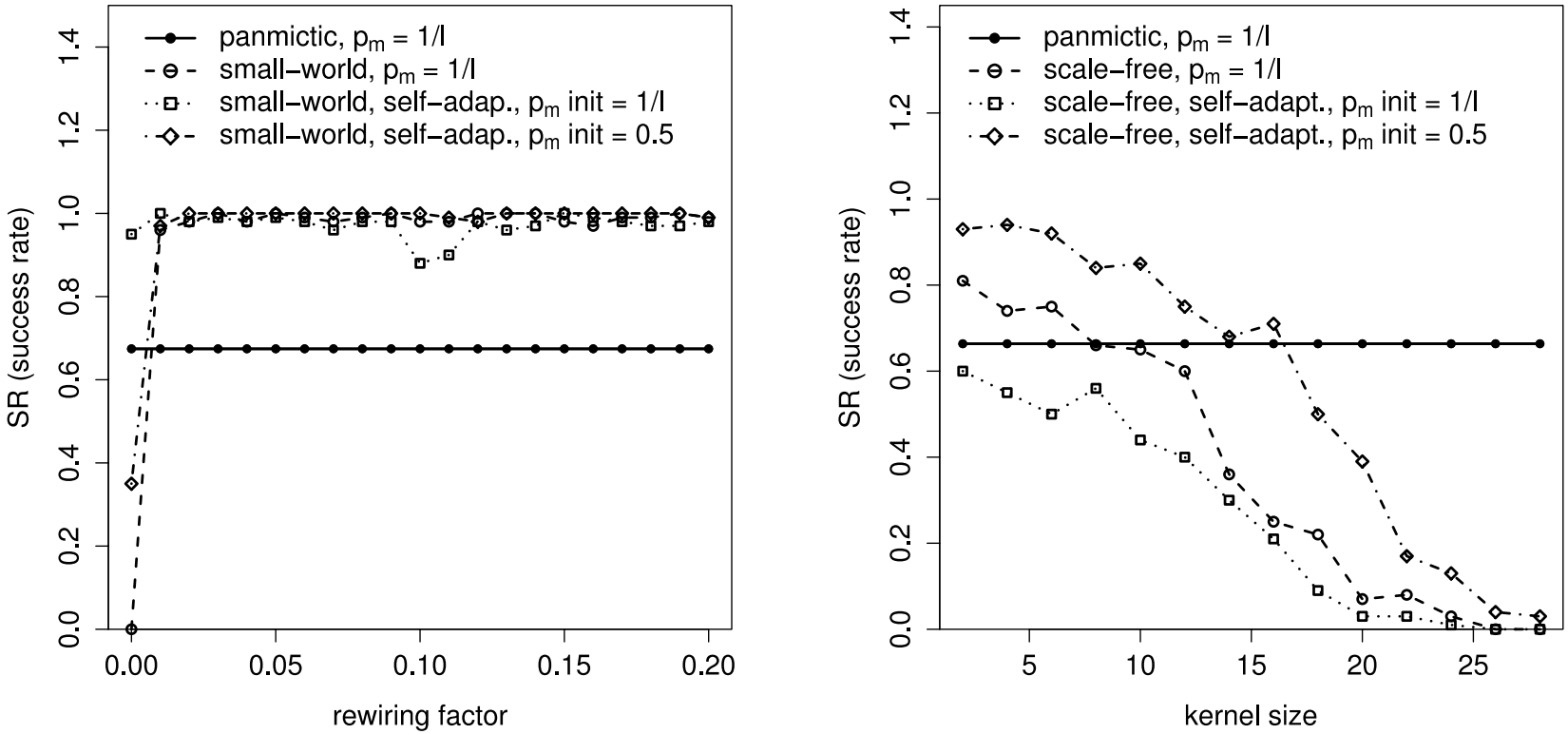
Takeover Times on Barabási–Albert Networks



Left: best individual uniformly distributed; Right: best starts in a hub.

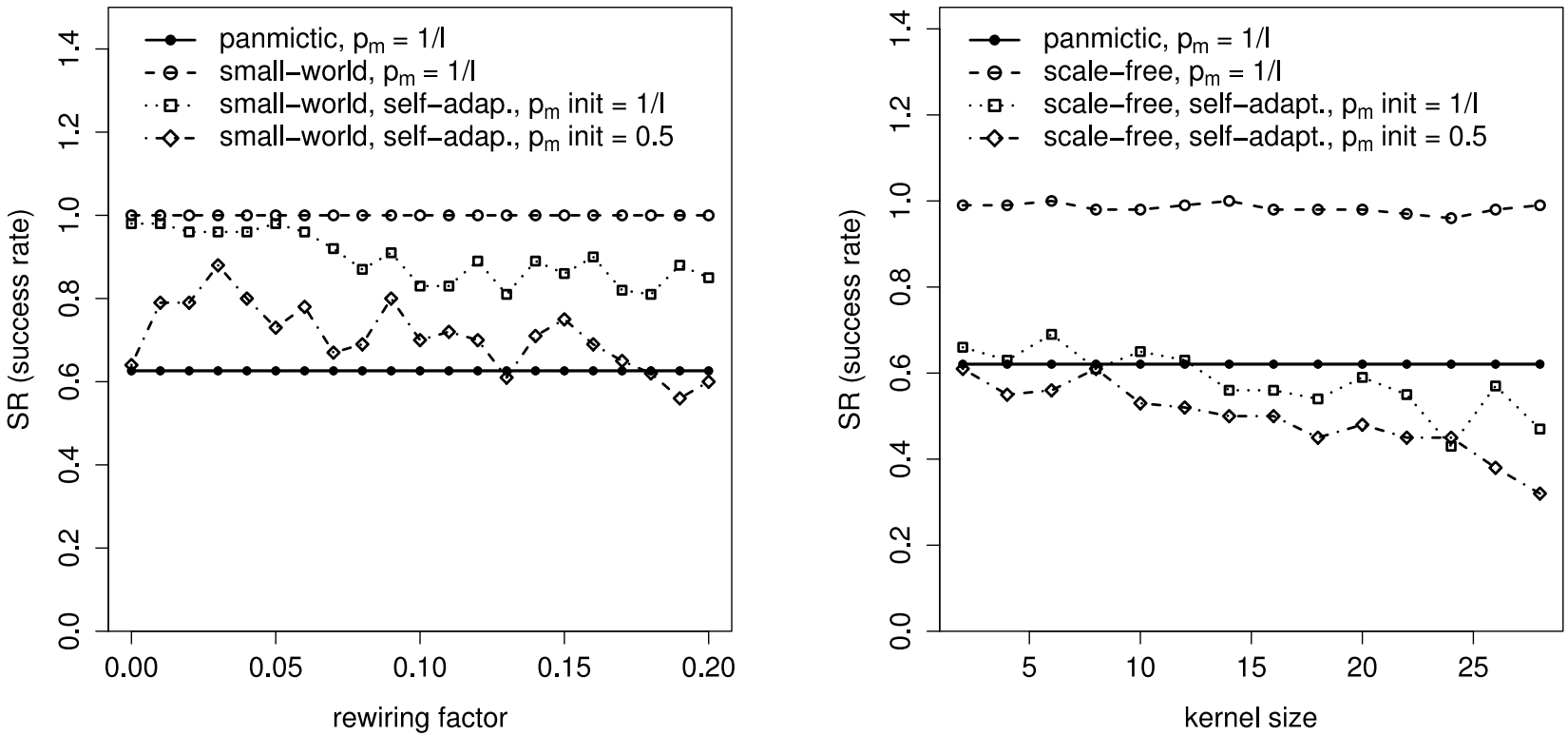
Propagation of the best is extremely fast, especially when the original individual sits on a hub

Some Experimental Results on Discrete Problems I



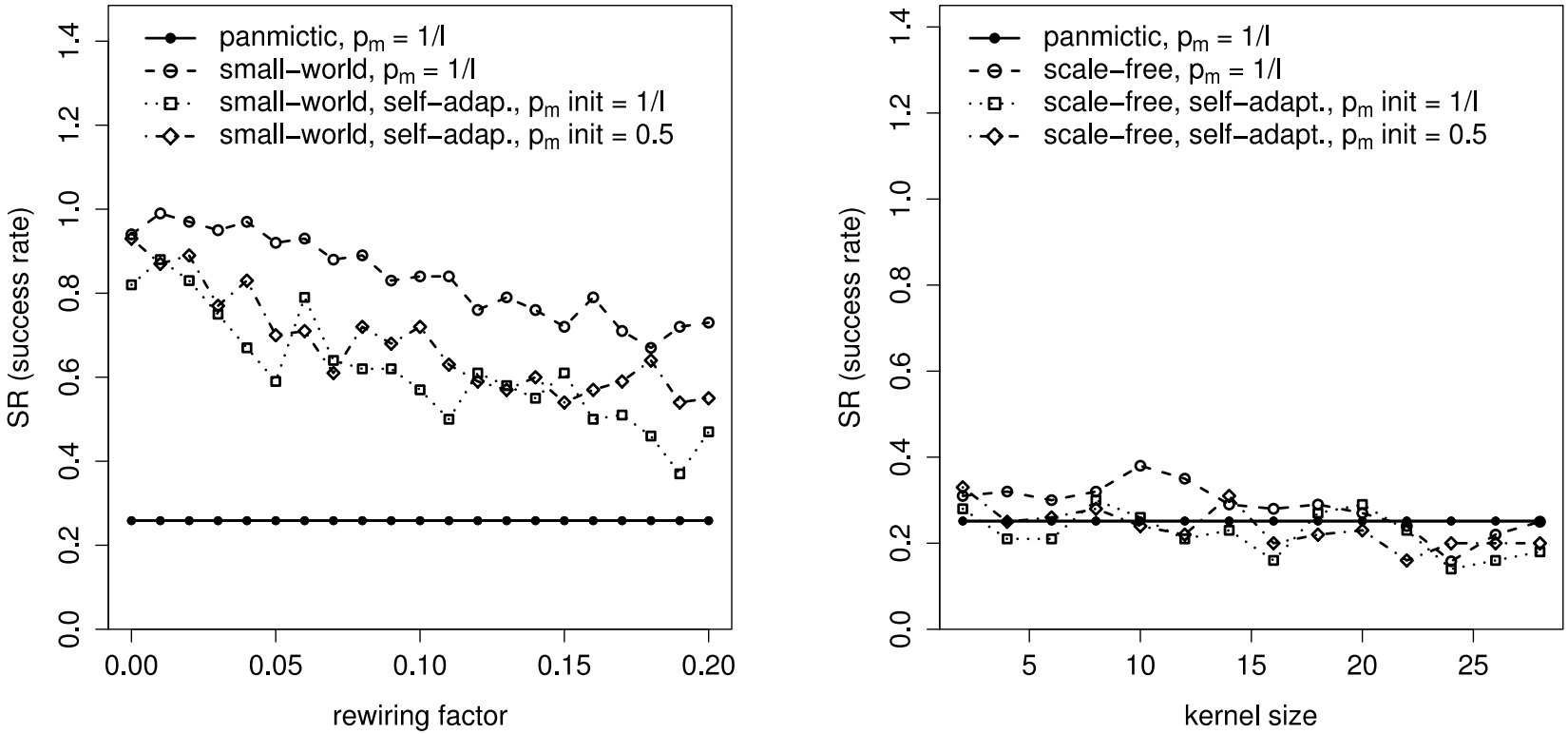
Success rates for small-world (left) and scale-free (right) topology on the MMDP problem. Each point is the average of 100 runs

Some Experimental Results on Discrete Problems II



Success rates for small-world (left) and scale-free (right) topology on the ECC problem. Each point is the average of 100 runs

Some Experimental Results on Discrete Problems III



Success rates for small-world (left) and scale-free (right) topology on the P-PEAKS problem. Each point is the average of 100 runs

Summary of Results on Static Complex Networks

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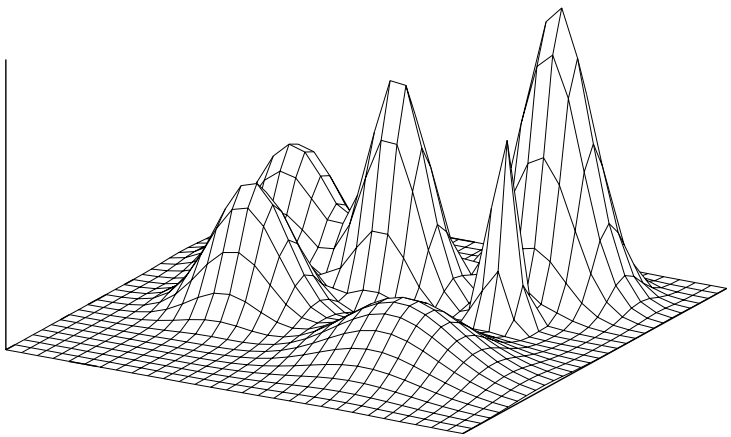
- Scale-free structured populations do not seem to help much: they converge too fast, even when the initial clique is small
- Dynamically varying population topologies might be the answer

Some references for this part

1. E. Alba and B. Dorronsoro, Cellular Genetic Algorithms, Springer, 2008
2. M. Tomassini, Spatially Structured Evolutionary Algorithms, Springer, 2005
3. M. Giacobini, M. Tomassini, A. Tettamanzi, Takeover time curves in random and small-world structured populations, GECCO '05, ACM Press , 1333-1340, 2005.
4. M.Giacobini, M. Preuss, M. Tomassini, Effects of scale-free and small-world topologies on binary-coded, self-adaptive CAS, EvoCop 2006, Lecture Notes in Computer Science, 3906, 86-98, Springer, 2006.

More recent work in this direction, including dynamical and self-organized networked populations: J. Payne, J.-L. Jiménez-Laredo and cw, Whitacre and cw, plus a few other groups

Discrete Search Spaces



Fitness landscape $(\mathcal{S}, \mathcal{V}, f)$:

- $\mathcal{S}$: set of admissible solutions,
- $\mathcal{V} : \mathcal{S} \rightarrow 2^{\mathcal{S}}$: neighborhood function,
- $f : \mathcal{S} \rightarrow \mathbb{R}$: fitness function.

Basin of Attraction

Hill-Climbing (HC) algorithm

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Choose initial solution $s \in S$
repeat
  choose $s^{'} \in \mathcal{V}(s)$ such that $f(s^{'}) = max_{x \in \mathcal{V}(s)}\ f(x)$
  if $f(s) < f(s^{'})$ **then**
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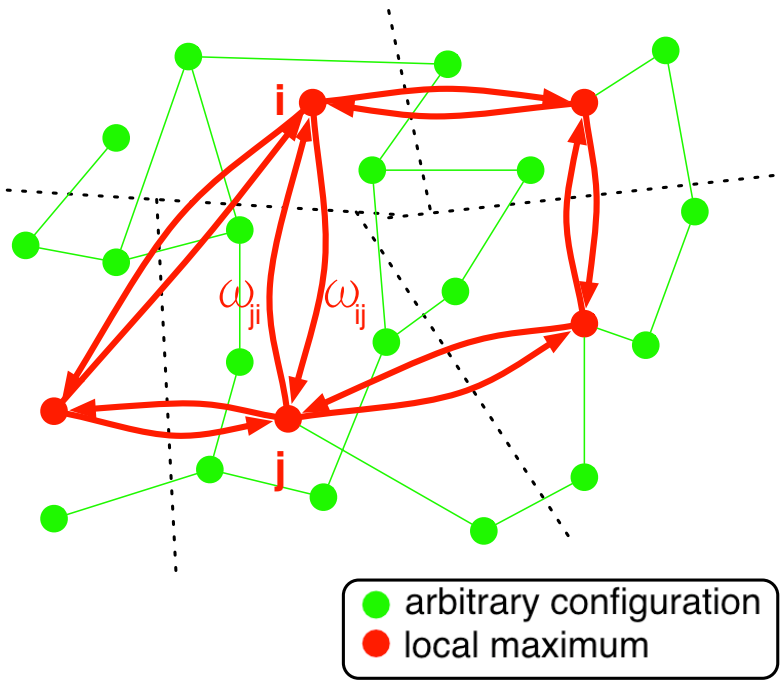
Basin of attraction of s^* :

$$\{s \in \mathcal{S} \mid HillClimbing(s) = s^*\}.$$

Local Optima Network (LON)



Nodes : set of local optima $\mathcal{S}^*$
Edges : notion of connectivity between basins

- e_{ij} between i and j if there is at least a pair of neighbours s_i and $s_j \in \mathcal{V}(s_i)$ such that $s_i \in b_i$ and $s_j \in b_j$
- weights w_{ij} attached to edges $\longrightarrow$ transition probabilities between basins

Underlying Ideas and Goals

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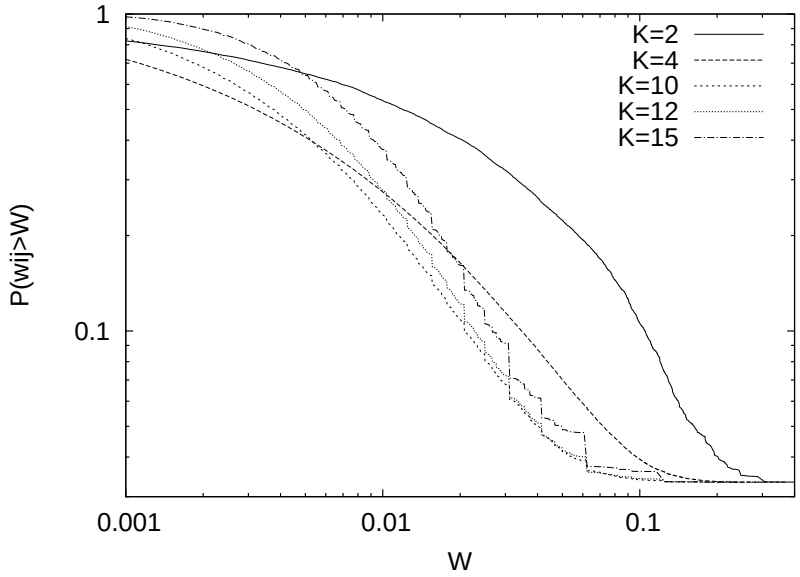
- Bring the tools of *complex networks* analysis to the study of the structure of combinatorial fitness landscapes
  - Relate problem features such as fitness distribution, basins number and size distribution etc. with network structure
  - Use network information to design more effective heuristic search algorithms

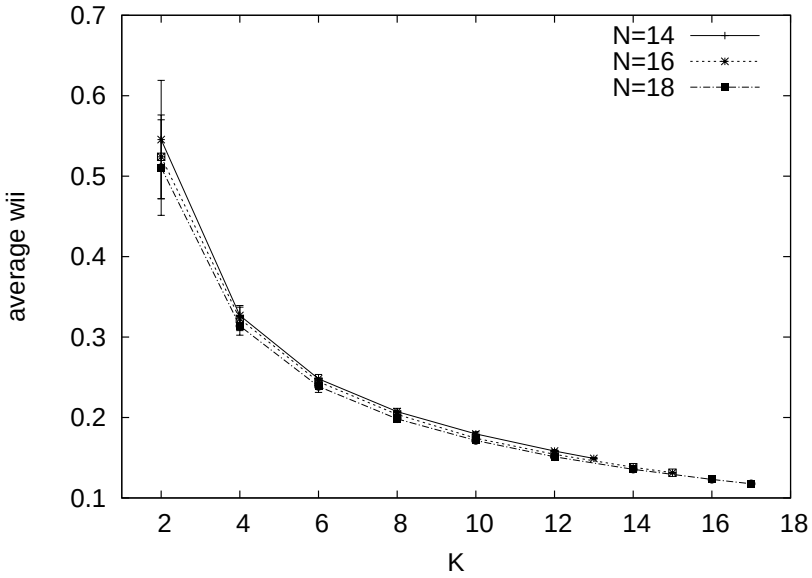
The *NK* Landscape Case: Outgoing weight distribution



Cumulative distribution of the network weights w_{ij} for outgoing edges with $j \neq i$ in log-log scale, $N = 16$

- Weights (transition prob. to neighbouring basins) are small
- The distributions are not uniform or Poissonian, nor power laws
- For high K the decay is faster
- Low K has longer tails (on average the transition probabilities are higher for low K)

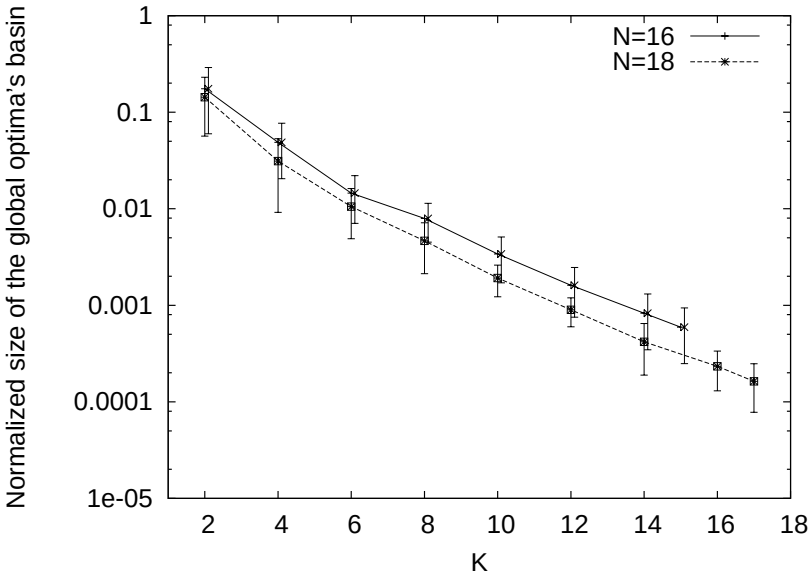
Average weights to remain in the same basin



Average weight w_{ii} according to the parameter N and K

- Weights to remains in the same basin w_{ii}, are large compared to w_{ij} with $i \neq j$
- w_{ii} are higher for low K (50% for $K = 2$, above 12% for high K),
- It seems easier to leave the basin for high K (high exploration), however, number the of local optima increases fast with K

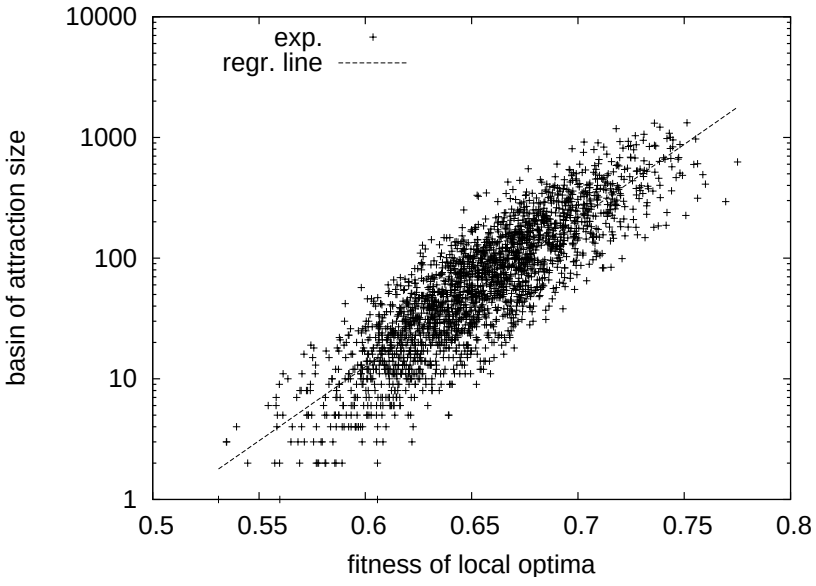
Global optimum basin size



Size of the basin corresponding to the global maximum for each K

- Trend: the basin shrinks very quickly with increasing K.
- For higher K, it is more difficult for a search algorithm to locate the basin of attraction of the global optimum

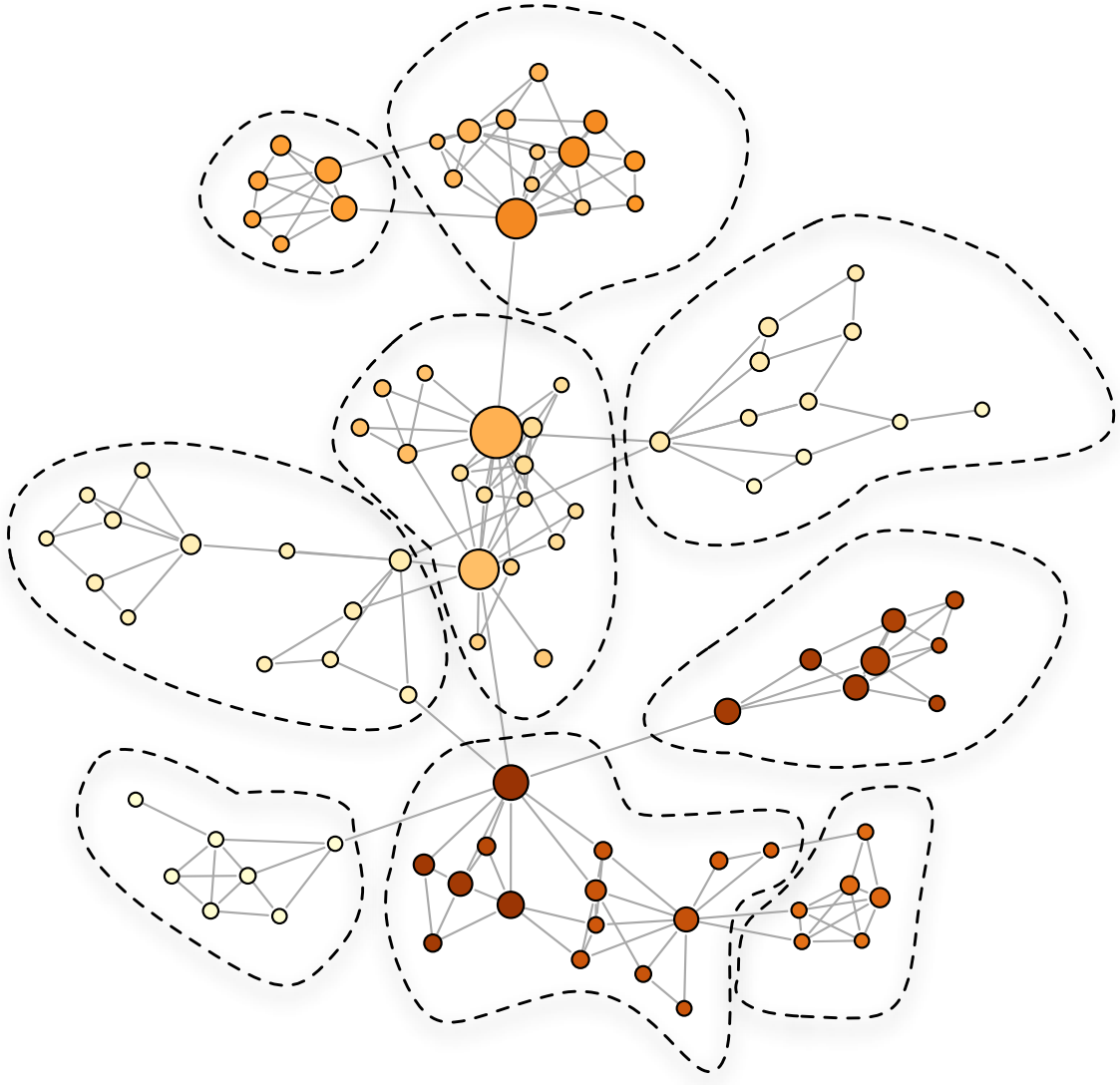
Fitness vs. basin size



Fitness of local optima vs. their corresponding basins sizes

- Trend: clear positive correlation between the fitness values of maxima and their basins' sizes
- On average, the global optimum seems easier to find than another local optimum, however, the number of local optima increases exponentially with increasing K

Community Structure in the LON of a Small Real-Like Instance of QAP



Some references for this part

1. G. Ochoa, M. Tomassini, S. Vérel, C. Darabos, A Study of NK Landscapes' Basins and Local Optima Networks, GECCO '08, ACM Press, 555-562, 2008.
2. M. Tomassini,S. Vérel, G. Ochoa, Complex Networks Analysis of Combinatorial Spaces: the NK Landscape Case, Phys. Rev. E, 78, 6, 066114, 2008.
3. S. Vérel, G. Ochoa, M. Tomassini, Local Optima Networks of NK Landscapes with Neutrality, IEEE Transactions on Evolutionary Computation, to appear, 2011.
4. F. Daolio, M. Tomassini, S. Vérel, G. Ochoa, Communities of Minima in Local Optima Networks of Combinatorial Spaces, Physica A, 390, 1684-1694, 2011.