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Analysing the Robustness of Multiobjectivisation Approaches Applied to Large Scale Optimisation Problems

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Outline

State of the Art (I)

- Many real world problems require the application of optimisation strategies.
- The usage of exact approaches is unaffordable.
- One of the most popular approximated techniques is the family of metaheuristics.
- Among them, EAs have provided good results when they have been applied to mono-objective and multi-objective optimisation problems.
- Several components must be specified in order to entirely define an EA:
    - Survivor selection mechanism
    - Parent selection strategy
    - Genetic operators
    - etc.

State of the Art (II)

- EAs may have a tendency to converge towards local optima.
- In order to deal with stagnation, several methods have been designed:
  - Restarting the algorithm when stagnation is detected.
  - Using a random component along the search.
  - Maintaining some memory.
  - etc.
- In the particular case of an EA:
  - Increasing the mutation rate.
  - Using selection mechanisms that maintain a diverse population.
  - Applying *Multiobjectivisation*.

Multiobjectivisation

- Multiobjectivisation transforms a mono-objective optimisation problem in a multi-objective one.
- By the use of multiobjectivisation, the fitness landscape is modified.
- Using multiobjectivisation can make easier the resolution of a problem.
- However, it can also produce a harder problem.
- There are different ways to multiobjectivise a problem:
  - Decomposition of the original objective.
  - Aggregation of new objectives:
    - Considering problem-dependent information.
    - Considering problem-independent information.

Objectives of the Research

- Analysing the validity and the robustness of a hybridisation among multiobjectivisation techniques and EAs.
- Demonstrating the advantages and drawbacks of multiobjectivisation.
- Improving the results obtained by the best currently known algorithms is NOT an objective of this research.

Description

- Different analyses have been carried out taking into consideration a set of mono-objective scalable continuous optimisation problems.
- The test suite was proposed for the "*Special Issue of Soft Computing on Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems*".
- The analyses have been applied to the problems F1 - F11.
- These problems combine different properties:
  - Modality
  - Separability
  - Ease of optimisation dimension by dimension

Features

Function	Name	Range	Fitness Optimum
F1	Shifted Sphere Function	$[-100, 100]^D$	-450
F2	Shifted Schwefel's Problem 2.21	$[-100, 100]^D$	-450
F3	Shifted Rosenbrock's Function	$[-100, 100]^D$	390
F4	Shifted Rastrigin's Function	$[-5, 5]^D$	-330
F5	Shifted Griewank's Function	$[-600, 600]^D$	-180
F6	Shifted Ackley's Function	$[-32, 32]^D$	-140
F7	Shifted Schwefel's Problem 2.22	$[-10, 10]^D$	0
F8	Shifted Schwefel's Problem 1.2	$[-65536, 65536]^D$	0
F9	Shifted Extended f_{10}	$[-100, 100]^D$	0
F10	Shifted Bohachevsky	$[-15, 15]^D$	0
F11	Shifted Schaffer	$[-100, 100]^D$	0

Properties

Function	Unimodal/ Multimodal	Shifted	Separable	Easily optimised dimension by dimension
F1	U	Y	Y	Y
F2	U	Y	N	N
F3	M	Y	N	Y
F4	M	Y	Y	Y
F5	M	Y	N	N
F6	M	Y	Y	Y
F7	U	Y	Y	Y
F8	U	Y	N	N
F9	U	Y	N	Y
F10	U	Y	N	N
F11	U	Y	N	Y

Multiobjectivised Strategies

- Multiobjectivisation of the benchmark problems has been performed by adding a new function as the second objective to optimise:
  - DCN: Euclidean Distance to the closest individual.
  - ADI: Average Euclidean Distance to all individuals.
  - DBI: Euclidean Distance to the individual with the lowest fitness.
- The aforementioned functions have considered the Euclidean Distance in the decision space.
- NSGA-II has been applied to the multiobjectivised versions of the problems.

Mono-objective Strategies

- In addition, a set of mono-objective EAs has been applied to the mono-objective versions of the problems.
- Each mono-objective EA has implemented a different survivor selection operator:
  - Steady-State (SS): If the generated offspring is better than any of the individuals of the population, the worst of them is replaced by this new offspring.
  - Generational with Elitism (GEN): In each generation, all parents, except the fittest one, are discarded, and they are replaced by the generated offsprings.
  - Replace Worst (RW): The fittest individuals, among parents and offsprings, are selected to survive.

Other Components

- The multiobjectivised and the mono-objective approaches have also used the following components:
  - A direct encoding of the individuals has been applied.
  - The parent selection mechanism has been the well-known *Binary Tournament*.
  - The *Uniform Mutation* (UM) operator has been applied with a probability p_m.
  - The *Simulated Binary Crossover* (SBX) operator has been applied with a probability p_c.

Computational Resources and Parameterisation

- Tests have been run on a machine with the following characteristics:
  - Debian GNU/Linux
  - 4 AMD ® Opteron ™ (6164 HE) at 1.7 GHz
  - 64 GB RAM
- The compiler has been *gcc 4.4.5*.
- The following parameterisation has been used:
  - *Number of variables D*: 50 and 500
  - *Population size n*: 5, 10, and 20 individuals
  - *Stopping Criterion*: $5000 \cdot D$ evaluations
  - $p_m = \frac{1}{D}$, and $p_c = 1$.
- Each execution has been repeated 30 times.
- Statistical tests have considered a confidence level of 95%.

First Analysis

- *Objective*: Discovering the best mono-objective and multiobjectivised approaches for each problem.
- The median of the fitness achieved at the end of each execution has been considered.
- The superiority of GEN and DCN approaches is clear.

First Analysis - Best Approaches ($D = 50$)

	F1	F2	F3	F4	F5	F6
Mono_5	GEN	GEN	GEN	GEN	GEN	GEN
Mono_10	GEN	GEN	GEN	GEN	GEN	GEN
Mono_20	GEN	GEN	GEN	RW	GEN	GEN
Multi_5	DCN	DCN	DCN	DCN	DCN	DCN
Multi_10	DCN	DCN	DCN	DCN	DCN	DCN
Multi_20	DCN	DCN	DCN	DCN	DCN	DCN

	F7	F8	F9	F10	F11
Mono_5	GEN	GEN	GEN	GEN	GEN
Mono_10	GEN	GEN	GEN	GEN	GEN
Mono_20	GEN	GEN	GEN	GEN	GEN
Multi_5	DCN	ADI	DCN	DCN	DCN
Multi_10	DCN	DCN	DCN	DCN	DCN
Multi_20	DCN	DCN	DCN	DCN	DCN

First Analysis - Best Approaches ($D = 500$)

	F1	F2	F3	F4	F5	F6
Mono_5	GEN	RW	GEN	GEN	GEN	GEN
Mono_10	GEN	RW	GEN	GEN	GEN	GEN
Mono_20	GEN	RW	GEN	GEN	SS	GEN
Multi_5	DCN	ADI	DCN	DCN	DCN	DCN
Multi_10	DCN	DCN	DCN	DCN	DCN	DCN
Multi_20	DCN	DCN	DCN	DCN	DCN	DCN

	F7	F8	F9	F10	F11
Mono_5	GEN	GEN	GEN	GEN	GEN
Mono_10	GEN	GEN	GEN	GEN	GEN
Mono_20	GEN	GEN	GEN	GEN	GEN
Multi_5	DCN	DCN	DCN	DCN	DCN
Multi_10	DCN	DCN	DCN	DCN	DCN
Multi_20	DCN	DCN	DCN	DCN	DCN

Second Analysis

- *Objective*: Comparing the mono-objective and multiobjectivised approaches in terms of the achieved fitness.
- The median of the error achieved by the best mono-objective and multiobjectivised strategies has been considered.
- Moreover, a statistical analysis of the best mono-objective EA and the best multiobjectivised approach has been performed.

Second Analysis - Median of the Error ($D = 50$)

	F1	F2	F3	F4	F5	F6
Mono_5	0	1.76e+00	1.55e+02	0	2.25e-02	3.00e-03
Multi_5	0	**5.67e-01**	1.55e+02	0	**1.10e-02**	**1.00e-03**
Mono_10	**0**	1.63e+00	3.44e+02	0	1.50e-02	**2.00e-03**
Multi_10	5.00e-04	**1.05e+00**	**1.52e+02**	1.00e-03	1.00e-02	4.00e-03
Mono_20	6.00e-03	2.01e+00	3.38e+02	8.60e-02	2.10e-02	1.60e-02
Multi_20	**3.00e-03**	**1.64e+00**	1.97e+02	**1.00e-03**	1.90e-02	**9.00e-03**

	F7	F8	F9	F10	F11
Mono_5	4.57e-03	1.44e+09	7.24e+00	1.84e-04	6.80e+00
Multi_5	**1.37e-03**	**4.50e+08**	**4.65e+00**	**3.11e-05**	**4.46e+00**
Mono_10	2.59e-03	1.02e+09	7.77e+00	**1.16e-04**	7.05e+00
Multi_10	2.55e-03	**6.49e+08**	**5.62e+00**	4.69e-04	**6.12e+00**
Mono_20	1.82e-02	1.34e+09	1.79e+01	5.40e-03	1.98e+01
Multi_20	**3.70e-03**	**8.77e+08**	**6.92e+00**	**1.47e-03**	**8.00e+00**

Second Analysis - Median of the Error ($D = 500$)

	F1	F2	F3	F4	F5	F6
Mono_5	3.00e-03	1.67e+01	1.32e+03	3.00e-03	8.00e-03	3.00e-03
Multi_5	**0**	**1.14e+01**	1.26e+03	3.00e-03	0	**1.00e-03**
Mono_10	**2.00e-03**	**1.41e+01**	1.47e+03	3.95e-02	0	**3.00e-03**
Multi_10	5.00e-03	1.81e+01	1.48e+03	1.00e+00	1.00e-03	4.00e-03
Mono_20	6.50e-02	**1.54e+01**	1.78e+03	9.28e-01	8.00e-03	1.70e-02
Multi_20	**3.70e-02**	2.23e+01	1.87e+03	**8.95e-02**	**4.50e-03**	**1.30e-02**

	F7	F8	F9	F10	F11
Mono_5	4.02e-02	1.93e+11	7.53e+01	1.84e-03	7.40e+01
Multi_5	**1.37e-02**	**1.41e+11**	**4.49e+01**	**3.46e-04**	**4.44e+01**
Mono_10	**2.57e-02**	1.83e+11	8.42e+01	**1.32e-03**	8.24e+01
Multi_10	2.99e-02	**1.58e+11**	**6.30e+01**	4.60e-03	**6.19e+01**
Mono_20	1.77e-01	2.04e+11	2.08e+02	6.24e-02	2.08e+02
Multi_20	**3.83e-02**	**1.73e+11**	**7.78e+01**	2.76e-02	**7.63e+01**

Second Analysis - Summary

- The superiority of the multiobjectivised strategies has been clearly demonstrated.
- From a total number of 66 statistical tests, in 41 of them the best multiobjectivised approach has been statistically superior.
- In 9 tests, the best mono-objective algorithm has been statistically superior.
- Fixing the population size, the best multiobjectivised approach has been statistically better in a 76.2% of the cases, when it has been applied to a unimodal problem.
- When a multimodal problem has been considered, this ratio has decreased to a 37.5%.

Third Analysis

- *Objective*: Quantifying the improvement achieved by using multiobjectivisation, in terms of the invested evaluations.
- The median of the number of evaluations required to achieve a certain fitness value, regardless of the population size, has been calculated for the best mono-objective and multiobjectivised approaches.
- The fitness value has been fixed so that all executions have been able to achieve it.

Third Analysis - Percentage of Saved Evaluations

	F1	F2	F3	F4	F5	F6
$D = 50$	26.1%	45.9%	40.0%	-28.9%	33.3%	32.6%
$D = 500$	26.9%	-7.7%	25.0%	-21.2%	29.4%	28.0%

	F7	F8	F9	F10	F11
$D = 50$	29.2%	45.5%	31.0%	23.5%	33.3%
$D = 500$	25.9%	38.7%	29.4%	35.3%	29.4%

Third Analysis - Summary

- The superiority of multiobjectivisation has also been demonstrated in terms of the invested number of evaluations.
- The best-behaved multiobjectivised strategy has provided benefits in 19 cases from a total number of 22.
- In the majority of the cases, the percentage of saved evaluations has been large (in some cases about a 45%).

Conclusions (I)

- EAs may have a tendency to converge towards local optima.
- Multiobjectivisation is a strategy that can be used to deal with stagnation.
- This work has presented a hybridisation among multiobjectivisation techniques and EAs.
- Several well-known scalable mono-objective benchmark problems have been multiobjectivised with different approaches based on the Euclidean distance.
- A set of mono-objective EAs has been applied to the mono-objective versions of the problems.
- The NSGA-II has been applied to the multiobjectivised versions of the problems.

Conclusions (II)

- The best-behaved mono-objective algorithm is based on using a generational survivor selection mechanism with elitism (GEN).
- The best-behaved multiobjectivisation strategy is based on using the Euclidean distance to the closest neighbour in the population (DCN).
- The advantages of multiobjectivisation, in terms of quality and saved resources, have been demonstrated by the experimental evaluation.
- The benefits have been more outstanding for the unimodal benchmark problems than for the multimodal ones.

Future Work

- Multiobjectivisation should be applied to other mono-objective benchmark problems.
- It would be interesting to test multiobjectivisation with other kind of metaheuristics.
- In addition, instead of using the Euclidean distance, other metrics should be applied to multiobjectivise the mono-objective benchmark problems.
- These tasks could help to identify alternative features of the landscape, which may influence the multiobjectivisation benefits.

Questions?

Thank you for your attention!

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