Probabilistic Real-Time Analysis

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Problem Statement

- Real-Time Systems: timing constraints to enforce and guarantee in all the conditions
- Task Scheduling: task execution through scheduling algorithms

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Real-Time System: $S = (A, R, \Gamma)$

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Scheduling Algorithms A

- Priority-based
- Static or Dynamic

Timing constraints...

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Resource R

- Computational resource
- Communication resource
- ...

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Real-Time System: $S = (A, R, \Gamma)$

Task set Γ

$$\Gamma = \{\tau_1, \tau_2, \ldots, \tau_n\}$$
$$\tau_i = (O_i, C_i, T_i, D_i)$$

Timing constraints $\rightarrow$ Deadline D_i

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WORST-CASE SYSTEM and ANALYSIS

Outline

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Real-Time Analysis: Abstractions

- *Workload bound function* wbf: the maximum amount of resource required by that task
- *Demand bound function* dbf: the minimum amount of resource demanded by that task in order to execute and meet its timing constraint
- *supply bound function* sbf: the minimum resource provisioning from a resource provisioning system element

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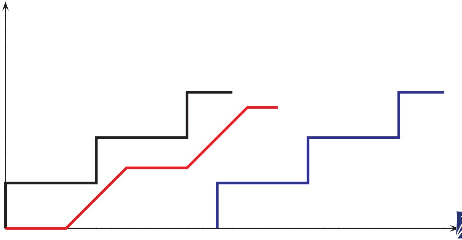
Functions



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Real-Time Analysis: Approximations

Approximating the resource supply sbf

$$\mathsf{bdf}(t) = \max\{0, \alpha(t - \Delta)\}$$

$$\alpha = \lim_{t \to \infty} \frac{\mathsf{sbf}(t)}{t}$$
$$\Delta = \inf\{q \mid \alpha(t - q) \leq \mathsf{sbf}(t)\ \forall t\}$$

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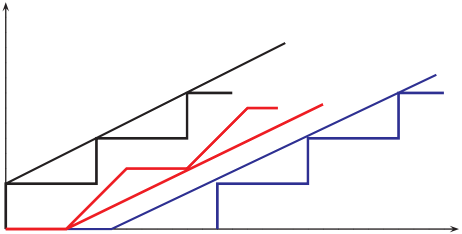

Real-Time Analysis: Schedulability

Schedulability

A real-time system is schedulable if all the tasks composing the system meet their deadline while executing

Scheduling conditions are defined by comparing the resource request (workload or resource demand) and the resource provisioning

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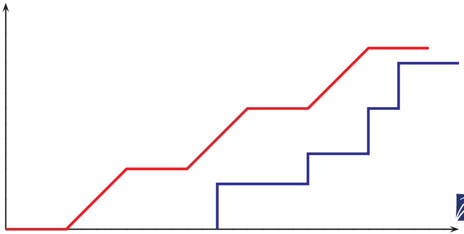
Comparison: timing guarantees (hard or soft)



Real-Time Analysis: an Example of Schedulability

i.e. *Earliest Deadline First (EDF) scheduling paradigm, a task set* Γ*, receiving an amount of resource* sbf *can be guaranteed schedulable (its deadline can be guaranteed) if and only if* $\forall t \quad \text{dbf}_\Gamma(t) \leq \text{sbf}(t)$

With the bounded-delay linear approximation, *the feasibility condition becomes a sufficient condition* $\forall\ t \quad \text{bdf}(t) \leq \text{sbf}(t)$

i.e. with α, Δ approximation

$$\forall t \in \mathrm{D} \ : \ \text{dbf}(t) \leq \alpha(t - \Delta)$$

D is the set of deadlines the application schedulability has to be checked

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Real-Time Analysis: Feasibility Space

Schedulability conditions also relate to schedulability regions in representation space

Example: the (α, Δ)-space - possible to define feasibility regions where the task set is schedulable with the (α, Δ) resource assignment

i.e. EDF, the application feasibility region is defined by
$\forall t \in \mathrm{D} \; : \; \mathrm{dbf}(t) \leq \alpha(t - \Delta)$

$$\forall t \in \mathrm{D} \; : \; \Delta \leq t - \frac{\mathrm{dbf}(t)}{\alpha}$$

$$\Delta \leq \min_{t \in \mathrm{D}} \left\{ t - \frac{\mathrm{dbf}(t)}{\alpha} \right\}$$

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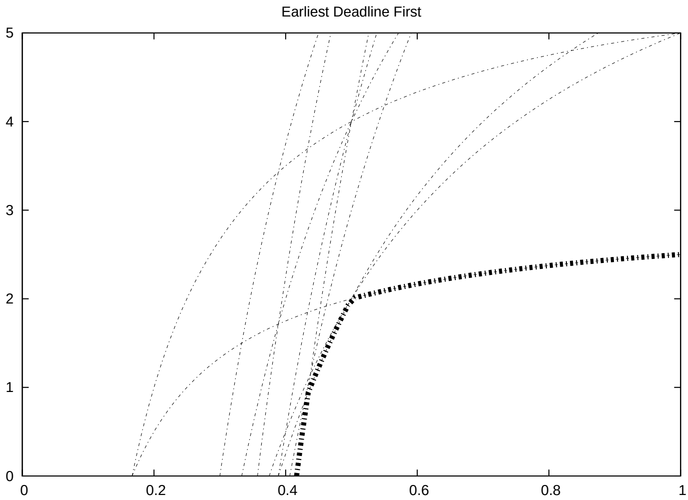
Real-Time Analysis: Feasibility Space



Outline

Motivations to Probabilities

- *Worst-case* timing analysis to validate the system

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HOWEVER, a worst-case analysis provides pessimistic results (WCET $\equiv$ pessimism) that not all real-time systemsems can afford

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The pessimism can be decreased by using probabilistic approaches.
What else?

- Reliability analysis, used to estimate the imperfection of reality
- Unreliable nature of the system environment and the system elements

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- Unreliable nature of the system environment and the system elements

Guarantee timing constraints needed for hard and soft real-time systems/applications

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Outline

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A Probabilistic Model

$$\Gamma = \{\tau_1, \tau_2, \ldots, \tau_n\}$$
$$\tau_i = (\mathcal{C}_i, D_i, T_i)$$

$\mathcal{C}_i$ - random variable on the execution time with a known probability function $f_{\mathcal{C}_i}(\cdot)$ ($f_{\mathcal{C}_i}(C) = P(\mathcal{C}_i = C)$)

$$\mathcal{C}_i = \begin{pmatrix} C_i^{\max} = C_i^0 & C_i^1 & \cdots & C_i^{\min} = C_i^m \\ f_{\mathcal{C}_i}(C_i^{\max}) & f_{\mathcal{C}_i}(C_i^1) & \cdots & f_{\mathcal{C}_i}(C_i^{\min}) \end{pmatrix}$$

$$\sum_j f_{\mathcal{C}_i}(C_i^j) = 1$$

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Probabilistic Analysis

Probabilistic functions and probabilistic approximation results in **probabilistic bounds to the system behaviour**

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Probabilistic dbf

$$\mathrm{dbf}_{i,j}(t) = \max\left\{0, \left(\left\lfloor \frac{t - D_i}{T_i} + 1 \right\rfloor C_i^j\right)\right\}$$

Each $\mathrm{dbf}_{i,j}$ has a bounding probability $p_{i,j} = 1 - \sum_{k \,:\, C_i^k \leq C_i^j} f_{\mathcal{C}_i}(C_i^k)$.

Probabilities: probability of bounding the resource demand

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Probabilistic functions: $\langle \mathrm{dbf}_{i,j}, p_{i,j} \rangle$

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Probabilistic sbf

Periodic service provisioning (Q, P), the worst-case/minimum resource supply $[0, t)$

$$\mathrm{sbf}(t) = \max\{0, (k-1)Q, t-(k+1)(P-Q)\}$$
$$k = \lceil \frac{t-(P-Q)}{P} \rceil$$

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Probabilistic functions:$\langle \mathrm{sbf}_k, p_k \rangle$

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Probabilistic Feasibility Space: an Example

Test Case

Given a probabilistic task $\tau = (0, \begin{pmatrix} 1 & 2 & 3 \\ 0.6 & 0.3 & 0.1 \end{pmatrix}, 10, 10)$, The possible probabilistic demand bound curves are

$$\text{dbf}_1 = \left\lfloor \frac{t-10}{10} + 1 \right\rfloor 1 \quad \rightarrow \quad (\alpha_1, \Delta_1)$$
$$\text{dbf}_2 = \left\lfloor \frac{t-10}{10} + 1 \right\rfloor 2 \quad \rightarrow \quad (\alpha_2, \Delta_2)$$
$$\text{dbf}_3 = \left\lfloor \frac{t-10}{10} + 1 \right\rfloor 3 \quad \rightarrow \quad (\alpha_3, \Delta_3)$$

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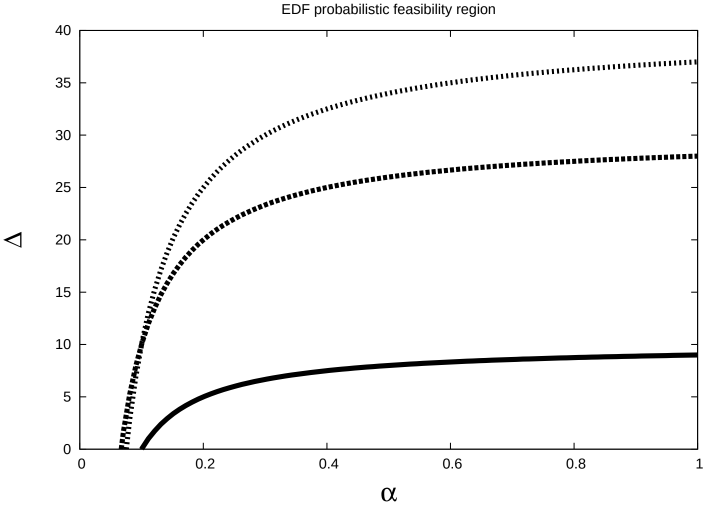
Probabilistic Feasibility Space: an Example



Remarks

Flexibility of the probabilistic model and analysis allows to:

- tackle with hard real-time
- easily extend to the common soft real-time case
- efficiently face many source of pessimism of the real-time analysis (WCET, approximations, etc.)

Increase the Complexity

With a complete probabilistic task model:

$$\tau_i = (\mathcal{C}_i, D_i, \mathcal{T}_i)$$

- $\mathcal{C}_i$ - random variable on the execution time with a known probability function $f_{\mathcal{C}_i}(\cdot)$ ($f_{\mathcal{C}_i}(c) = P(\mathcal{C}_i = c)$)
- $\mathcal{T}_i$ - random variable on the period with a known probability function $f_{\mathcal{T}_i}(\cdot)$ ($f_{\mathcal{T}_i}(T) = P(\mathcal{T}_i = T)$)

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$$\mathcal{T}_i = \begin{pmatrix} T_i^{\max} = T_i^0 & T_i^1 & \cdots & T_i^{\min} = T_i^m \\ f_{\mathcal{T}_i}(T_i^{\max}) & f_{\mathcal{T}_i}(T_i^1) & \cdots & f_{\mathcal{T}_i}(T_i^{\min}) \end{pmatrix}; \quad \sum_j f_{\mathcal{T}_i}(T_i^j) = 1$$

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Some Questions

1. How to derive conclusions from the probabilistic (α, Δ)-space?
2. What are other possible conclusions from the analysis of the $\alpha, \Delta)$-space?
3. How to study the probabilistic (α, Δ)-space in the general case? How to derive conclusions from the general case of probabilistic (α, Δ)-space?
4. Extend the model to other aspects of the real-time analysis?
5. A new model for probabilistic real-time?

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